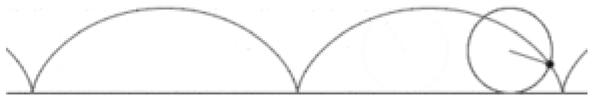
WIKIPEDIA

Trochoid

A **trochoid** (from the Greek word for wheel, "trochos") is the curve described by a fixed point on a circle as it rolls along a straight line.[1] The cycloid is a notable member of the trochoid family. The word "trochoid" was coined by Gilles de Roberval.



A cycloid (a common **trochoid**) generated by a rolling circle

Contents

Basic description

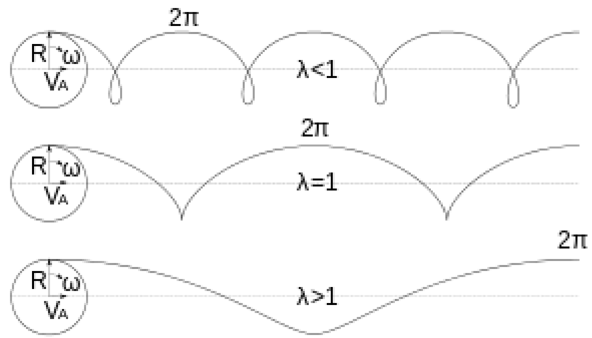
As a circle of radius a rolls without slipping along a line L, the center C moves parallel to L, and every other point P in the rotating plane rigidly attached to the circle traces the curve called the trochoid. Let $CP = b$. Parametric equations of the trochoid for which L is the x-axis are

$$x = a\theta - b\sin(\theta)$$
$$y = a - b\cos(\theta)$$

where θ is the variable angle through which the circle rolls.

Curtate, common, prolate

If P lies inside the circle ($b < a$), on its circumference ($b = a$), or outside ($b > a$), the trochoid is described as being curtate ("contracted"), common, or prolate ("extended"), respectively.[2] A curtate trochoid is traced by a pedal when a normally geared bicycle is pedaled along a straight line.[3] A prolate trochoid is traced by the tip of a paddle when a boat is driven with constant velocity by paddle wheels; this curve contains loops. A common trochoid, also called a cycloid, has cusps at the points where P touches the L.



From top to bottom a prolate, common and curtate trochoid respectively, with *b* being the same for all curves and with λ = *a* / *b*

General description

A more general approach would define a trochoid as the locus of a point (x, y) orbiting at a constant rate around an axis located at (x', y'),

$$x = x' + r_1 \cos(\omega_1 t + \phi_1),\ y = y' + r_1 \sin(\omega_1 t + \phi_1),\ r_1 > 0,$$

which axis is being translated in the x-y-plane at a constant rate in *either* a straight line,

$$x' = x_0 + v_{2x} t,\ y' = y_0 + v_{2y} t$$
$$\therefore x = x_0 + r_1 \cos(\omega_1 t + \phi_1) + v_{2x} t,\ y = y_0 + r_1 \sin(\omega_1 t + \phi_1) + v_{2y} t,$$

or a circular path (another orbit) around (x_0, y_0) (the hypotrochoid/epitrochoid case),

$$x' = x_0 + r_2 \cos(\omega_2 t + \phi_2),\ y' = y_0 + r_2 \sin(\omega_2 t + \phi_2),\ r_2 \geq 0$$
$$\therefore x = x_0 + r_1 \cos(\omega_1 t + \phi_1) + r_2 \cos(\omega_2 t + \phi_2),\ y = y_0 + r_1 \sin(\omega_1 t + \phi_1) + r_2 \sin(\omega_2 t + \phi_2),$$

The ratio of the rates of motion and whether the moving axis translates in a straight or circular path determines the shape of the trochoid. In the case of a straight path, one full rotation coincides with one period of a periodic (repeating) locus. In the case of a circular path for the moving axis, the locus is periodic only if the ratio of these angular motions, ω_1/ω_2, is a rational number, say p/q, where p & q are coprime, in which case, one period consists of p orbits around the moving axis and q orbits of the moving axis around the point (x_0, y_0). The special cases of the epicycloid and hypocycloid, generated by tracing the locus of a point on the perimeter of a circle of radius r_1 while it is rolled on the perimeter of a stationary circle of radius R, have the following properties:

$$\text{epicycloid:} \quad \omega_1/\omega_2 = p/q = r_2/r_1 = R/r_1 + 1,\ |p - q| \text{ cusps}$$
$$\text{hypocycloid:} \quad \omega_1/\omega_2 = p/q = -r_2/r_1 = -(R/r_1 - 1),\ |p - q| = |p| + |q| \text{ cusps}$$

where r_2 is the radius of the orbit of the moving axis. The number of cusps given above also hold true for any epitrochoid and hypotrochoid, with "cusps" replaced by either "radial maxima" or "radial minima."

See also

- Roulette (curve)
- List of periodic functions
- Epitrochoid
- Hypotrochoid
- Cycloid
- Cyclogon
- Spirograph
- Trochoidal wave

References

1. Weisstein, Eric W. "Trochoid" (http://mathworld.wolfram.com/Trochoid.html). *MathWorld*.
2. "Trochoid" (http://www.xahlee.org/SpecialPlaneCurves_dir/Trochoid_dir/trochoid.html). *Xah Math*. Retrieved October 4, 2014.
3. https://www.youtube.com/watch?v=aJhiY70KY5o

External links

- Online experiments with the Trochoid using JSXGraph (http://jsxgraph.uni-bayreuth.de/wiki/index.php/Trochoid)